

Infrared behavior and gauge artifacts in de Sitter spacetime

I. The photon field

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(Dated: November 10, 2010)

We study the infrared (long distance) behavior of the free photon field in de Sitter spacetime. Using a two-parameter family of gauge fixing terms, we show that the infrared (IR) behavior of the two-point function is highly gauge-dependent and ranges from vanishing to growing. This situation is in disagreement with its counterpart in flat spacetime, where the two-point function vanishes in the IR for any choice of the gauge fixing parameters. A criterion to isolate the “physical” part of the two-point function is given and is shown to lead to a well-behaved two-point function in the IR.

INTRODUCTION

The study of quantum field theory in the de Sitter background is of paramount importance to the understanding of the early universe as well as its present accelerated expansion. One of the most striking, yet poorly understood, aspects of de Sitter QFT is the so called *infrared problem*. The simplest and most serious manifestation of this problem being the non-vanishing of the correlation functions for largely - spacelike and timelike - separated points. This is for instance a well-known fact for the massless minimally coupled (mmc) scalar and the graviton fields. In this letter we study the IR behavior of the photon field in de Sitter space and show that it exhibits similar IR pathologies. More importantly we are able to show that these are purely gauge artifacts.

The organization of the letter is as follows. First we compute the correlation function using the R_λ gauges $\lambda(\nabla_a A^a)^2/2$. Then we generalize the computation to the larger family of higher derivative gauge fixing terms $\lambda(\nabla_a A^a)^2/2 + \xi R^2(\nabla_a A^a)\square(\nabla_a A^a)/2$. Finally we propose a decomposition of the two-point function into a physical and a gauge part and we show that the physical part is vanishing in the IR.

The d -dimensional de Sitter spacetime X_d of “radius” R can be identified with the real one-sheeted hyperboloid in the $d+1$ Minkowski spacetime M_{d+1}

$$X_d = \{x \in \mathbb{R}^{d+1}, \eta_{\mu\nu} x^\mu x^\nu = R^2\}$$

where $\eta_{\mu\nu}$ is the “mostly plus” flat metric $\text{diag}(-1, +1, \dots, +1)$. Let $\mu(x, x')$ denote the geodesic distance between the points x and x' and g_{ab} be the d -dimensional de Sitter (or d -dimensional Minkowski if $R \rightarrow \infty$) metric. We will compute the Wightman two-point function $W_{aa'}(x, x') = \langle \Omega | A(x) A(x') | \Omega \rangle$ in different gauges. In the previous formula $|\Omega\rangle$ designates the Euclidean (Bunch-Davies) vacuum state. This vacuum being de Sitter invariant, $W_{aa'}(x, x')$ is a

maximally symmetric bi-tensor. As shown in detail in [5], all such bi-tensors can be expressed in terms of the parallel propagator $g_{aa'}(x, x')$ between x and x' , and the two vectors $n_a = \nabla_a \mu$ and $n_{a'} = \nabla_{a'} \mu$. Our two-point function is thus written as

$$W_{aa'}(x, x') = \alpha_1(\mu) g_{aa'} + \alpha_2(\mu) n_a n_{a'}$$

where, for spacelike separations ($\mu^2 > 0$), α_1 and α_2 are scalar functions of the geodesic distance μ only. We will frequently use the following formulas [5]:

$$\begin{aligned} g_a^{b'} n_{b'} &= -n_{a'}, & g_{ab} &= g_a^{c'} g_{c'b} \\ \nabla_a n_b &= A(g_{ab} - n_a n_b), & \nabla_a n_{b'} &= C(g_{ab'} + n_a n_{b'}) \\ \nabla_a g_{bc'} &= -(A + C)(g_{ab} n_{c'} + g_{ac'} n_b) \end{aligned}$$

where

$$A = \frac{\cot(\mu/R)}{R}, \quad \text{and} \quad C = -\frac{\csc(\mu/R)}{R}.$$

It is also useful to introduce the quantity z given for spacelike separations ($\mu^2 > 0$ or $0 < z < 1$) by

$$z = \cos^2\left(\frac{\mu}{2R}\right). \quad (1)$$

All of our calculations are performed in the spacelike region $0 < z < 1$. They can be extended further by analytic continuation in the z variable to the cut complex plane $\mathbb{C} \setminus (1, \infty)$. Finally, the Feynman propagator $G_{aa'}^F = i\langle \Omega | T A_a(x) A_{a'}(x') | \Omega \rangle$, can be obtained as the limiting value of $W_{aa'}$ when approaching the branch cut $z > 1$ from above.

We end our introduction by noting that several, quite interesting, recent works (see [2] among many others) were devoted to the study of infrared effects in de Sitter. In these articles however, only massive (although *interacting*) fields are considered. IR pathologies are of course much stronger for massless fields and they appear already at the linear level.

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THE R_λ GAUGES

The gauge fixed action describing the massless vector field A_a is

$$S_\lambda = \int d^4x \sqrt{-g} \left(\frac{1}{4} F_{ab} F^{ab} + \frac{\lambda}{2} (\nabla_a A^a)^2 \right) \quad (2)$$

where $F_{ab} = \nabla_a A_b - \nabla_b A_a$. The resulting equation of motion is $D_{ab} A^b = 0$ where

$$D_{ab} = g_{ab} \square - \frac{(d-1)}{R^2} g_{ab} + (\lambda-1) \nabla_a \nabla_b. \quad (3)$$

Flat space. In flat space ($R \rightarrow \infty$), the Fourier transform of the Feynman propagator reads

$$G_{aa'}^F(k) = \frac{1}{k^2 - i\epsilon} g_{aa'} + \frac{1-\lambda}{\lambda} \frac{1}{(k^2 - i\epsilon)^2} k_a k_{a'}.$$

Using the (massless limit) of the scalar Feynman propagator (given, as always, for spacelike separations)

$$G^F(x, x') = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-x')}}{k^2 + m^2 - i\epsilon} = \frac{im}{(2\pi)^2} \frac{K_1(m\mu)}{\mu} \quad (4)$$

(where K is the modified Bessel function of the second kind) and its derivatives with respect to x and x' we get the coordinate-space two-point function

$$W_{aa'} = \left[\frac{\lambda+1}{8\pi^2 \lambda \mu^2} \right] g_{aa'} - \left[\frac{\lambda-1}{4\pi^2 \lambda \mu^2} \right] n_a n_{a'}. \quad (5)$$

For all values of λ , this expression vanishes for large time-like and spacelike separations as expected.

De Sitter space. Following [5], we write the two-point function in de Sitter space as

$$W_{aa'}^{[\lambda]}(x, x') = \alpha_1^{[\lambda]}(z) g_{aa'} + \alpha_2^{[\lambda]}(z) n_a n_{a'}. \quad (6)$$

where α_1, α_2 are scalar functions of the invariant quantity z defined in (1). The equation of motion $D_{ab} W_{a'}^b = 0$ implies two independent equations on α_1 and α_2 . These equations are most easily solved in the Feynman gauge $\lambda = 1$ and the result is found in [5] to be

$$\begin{aligned} \alpha_1^{[\lambda=1]}(z) &= \frac{1}{48\pi^2 R^2} \left[\frac{3}{1-z} + \frac{1}{z} + \left(\frac{2}{z} + \frac{1}{z^2} \right) \ln(1-z) \right] \\ \alpha_2^{[\lambda=1]}(z) &= \frac{1}{24\pi^2 R^2} \left[1 - \frac{1}{z} + \left(\frac{1}{z} - \frac{1}{z^2} \right) \ln(1-z) \right] \end{aligned}$$

Instead of vanishing for largely separated points, the correlation goes to a non-zero constant function

$$\lim_{z \rightarrow \infty} W_{aa'}^{[\lambda=1]}(x, x') = \frac{1}{24\pi^2 R^2} n_a n_{a'}. \quad (7)$$

This constant vanishes in the flat spacetime limit ($R \rightarrow \infty$) as expected. The non-vanishing of the correlator in de Sitter space seems to contradict our experience with the more familiar flat space QFT. It is indeed a general

result of the latter, sometimes referred to as the cluster decomposition principle [1], that the correlation functions must decay in the IR. To our knowledge, only very special QFT models, e.g confinement models like the massless scalar in two-dimensional flat space, seem to escape this result.

As explained in the introduction, it turns out that this IR pathological behavior of the photon field is typical of de Sitter massless fields. The most notable examples are the mmc scalar and the graviton fields. The correlation functions of these fields exhibit even stronger pathologies as they grow in the IR (see for instance [3, 4, 7, 8]).

This ‘‘IR problem’’ is at the heart of an open discussion in the community. It led some authors [6] to speak of a ‘‘striking violation of cluster decomposition properties of the de Sitter invariant vacuum state’’. It has also been shown that this growing of the graviton two-point function render scattering amplitudes divergent [10]. As a consequence, the authors of [6], among many others, suspect a *quantum instability* of de Sitter space, meaning that the de Sitter geometry is not a stable ground state of quantum gravity with a cosmological constant.

Other workers in the field, mainly motivated by the fact that the correlations of *some* gauge invariant quantities fall-off in the IR limit, claim that the non-vanishing of the two-point functions in the IR is nothing but a gauge artifact. For instance, the electromagnetic field correlation function is well-behaved in the IR [5]:

$$\langle F_{ab} F^{a'b'} \rangle = \frac{1}{8\pi^2 R^4} \frac{1}{(1-z)^2} \left(g_{[a}^{[a'} g_{b]}^{b']} + 4 n_{[a} g_{b]}^{[b'} n^{a']} \right).$$

The same is true for the graviton field as there is no growing terms in the correlation function of the Riemann tensor $\langle R_{abcd}(x) R_{a'b'c'd'}(x') \rangle$.

In the following we will study in detail the IR problem in the case of the photon field in de Sitter. The IR behavior of this field received considerably less attention in the literature than its counterpart for the mmc and the graviton field. This is certainly due to the importance of these two-fields in inflation theory. It is also probably due to the fact that in actual QED calculations, the non vanishing term (7) will not contribute to scattering processes (see last section). This model will however prove to be precious in realizing that - at least for some situations - serious IR pathologies in de Sitter space are - without doubt - gauge artifacts.

The equation of motion (3) applied to the two-point function (6) leads after some algebra to a coupled system of differential equations. Hereafter we will only need one of these equations, namely:

$$\begin{aligned} \mathbf{H} \left[1, d-2, \frac{1}{2} + \frac{1-\lambda}{2} \right] \alpha_1 + \frac{\lambda-1}{2} \alpha'_2 \\ + \frac{\lambda-\lambda_d}{\lambda_d-1} \left[\frac{\alpha_1}{z} + \frac{1}{2} \left(\frac{1}{1-z} - \frac{1}{z} \right) \alpha_2 \right] = 0 \end{aligned} \quad (8)$$

where the prime denote derivation with respect to the

variable z , $\lambda_d = \frac{d-3}{d-1}$ and \mathbf{H} is the hypergeometric operator defined by

$$\mathbf{H}[a, b, c] = z(1-z) \frac{d^2}{dz^2} + [c - (a+b+1)z] \frac{d}{dz} - ab$$

Another, moreover first order, relation between α_1 and α_2 is obtained by considering the two-point function of the field strength

$$\langle F_{ab} F^{a'b'} \rangle = \beta_1 g_{[a}^{[a'} g_{b]}^{b']} + \beta_2 n_{[a} g_{b]}^{[b'} n^{a']}. \quad (9)$$

In fact, the equation of motion (in vacuum) $\nabla^a F_{ab} = 0$ implies the relation [5]

$$\alpha_2 = \frac{z}{8\pi^2 R^2(z-1)} + 2(1-z)(z\alpha'_1 + \alpha_1) \quad (9)$$

We now specialize our results to the $d = 4$ case for simplicity. The general solution in the Euclidean vacuum is found to be

$$\begin{aligned} \alpha_1^{[\lambda]}(z) &= \frac{1}{48\pi^2 R^2 \lambda} \left[\frac{3(\lambda+1)}{2(1-z)} + \frac{3\lambda-1}{2z} + \right. \\ &\quad \left. (3\lambda-1) \left(\frac{1}{z} + \frac{1}{2z^2} \right) \ln(1-z) \right] \\ \alpha_2^{[\lambda]}(z) &= \frac{1}{24\pi^2 R^2 \lambda} \left[1 - \frac{3(\lambda-1)}{2(1-z)} - \frac{3\lambda-1}{2z} + \right. \\ &\quad \left. (3\lambda-1) \left(\frac{1}{2z} - \frac{1}{2z^2} \right) \ln(1-z) \right] \end{aligned}$$

and the IR behavior is given by

$$\lim_{z \rightarrow \pm\infty} W_{aa'}^{[\lambda]}(x, x') = \frac{1}{24\pi^2 R^2 \lambda} n_a n_{a'}. \quad (10)$$

This result means that one can cure any IR bad behavior by going to the *Landau gauge* $\lambda \rightarrow \infty$. More importantly, we see that the IR behavior of the correlation function is *pure gauge*, thus asking whether or not the two-point function is well or ill-behaved in the IR is in itself -at least partially-a misleading question.

Finally we note that there exists a special choice of the gauge fixing parameter, namely $\lambda = \frac{1}{3}$ (more generally λ_d in d dimensions), that cancels logarithmic terms and gives a particularly simple two-point function. We note, perhaps as a curiosity for the time being, that for large dimensions d , λ_d goes to the Feynman gauge $\lambda = 1$, which is the simplest gauge in flat space.

MORE PATHOLOGICAL GAUGES

We now consider a larger family of gauge fixing terms. Our motivation is that the IR pathology we exhibited in the last section, namely that the correlation function tends to a constant, might seem -I believe mistakenly-mild enough not to worry about. We will show that in

this more general gauge, the two-point function is actually growing in the IR, exactly like the mmc scalar or the graviton fields. We consider the action

$$S_{\lambda, \xi} = S_\lambda + \frac{\xi}{2m^2} \int d^4x \sqrt{-g} (\nabla_a A^a) \square (\nabla_a A^a) \quad (11)$$

where we have added a higher derivative gauge fixing term. Similar higher derivative gauge fixing are occasionally used in QCD and the electro-weak theories. They were also used to study flat space perturbative quantum gravity [9]. We note that if we want λ and ξ to be dimensionless, such higher derivative gauge fixing terms require the introduction of a mass quantity m . In flat space m is introduced by hand, while in de Sitter space the inverse of the de Sitter radius plays naturally this role: $m^2 = \frac{1}{R^2}$. This observation renders the introduction of higher derivative gauge fixing somehow more natural in the de Sitter case. The equation of motion reads

$$\left[\square g_{ab} + (\lambda - 1) \nabla_a \nabla_b + \frac{\xi}{m^2} \square \nabla_a \nabla_b \right] A^b = 0 \quad (12)$$

Flat space. The flat space Feynman propagator is found in Fourier space to be

$$\begin{aligned} \tilde{G}_{aa'}^F(k) &= \frac{g_{aa'}}{k^2 - i\epsilon} + \left[\frac{1 - \lambda}{\lambda(k^2 - i\epsilon)^2} \right. \\ &\quad \left. + \frac{\xi}{m^2 \lambda^2} \left(\frac{1}{k^2 - i\epsilon} - \frac{1}{k^2 - \frac{m^2 \lambda}{\xi} - i\epsilon} \right) \right] k_a k_{a'} \end{aligned}$$

The reader will note the relative negative sign typical of higher derivative theories in front of the last propagator. Using (4) and its derivatives the coordinate-space two-point function is then found to be

$$\begin{aligned} W_{aa'} &= \left[\frac{1}{2\pi^2 m^2 \lambda \mu^4 \nu^2} + \frac{1}{2\pi^2 m^2 \lambda \mu^4 \nu^2} + \frac{K_2(im\mu\nu)}{4\pi^2 \lambda \mu^2} \right] g_{aa'} + \\ &\quad \left[\frac{2}{\pi^2 m^2 \lambda \mu^4 \nu^2} + \frac{1 - \lambda}{4\pi^2 \lambda \mu^2} + \frac{K_0(im\mu\nu)}{\pi^2 \lambda \mu^2} \right. \\ &\quad \left. + i \left(\frac{m\nu}{4\pi^2 \lambda \mu} - \frac{2}{\pi^2 m \lambda \mu^3 \nu} \right) K_1(im\mu\nu) \right] n_a n_{a'} \end{aligned}$$

where we have introduced the quantity $\nu = (\lambda/\xi)^{1/2}$. For all values of λ and ν , this expression vanishes for large timelike and spacelike separations as expected. The short distance singularity is given by

$$W_{aa'} \sim \frac{1}{8\pi^2 \mu^2} (g_{aa'} - 2n_a n_{a'}).$$

We emphasize that the short distance singularity of the two theories (2) and (11) are different. This is due to the non commutation of the limits $\xi \rightarrow 0$ and $\mu \rightarrow 0$.

De Sitter space. We now consider the action (11) in de Sitter space and thus set $1/m^2 = R^2$. Following the same methods described above we obtain the following

equations on α_1 :

$$\alpha_1^{(4)} = \frac{4(\tau-2)(\tau+3)}{(z-1)^2 z^2} \alpha_1 + \frac{3(2z-1)(\tau^2+\tau-16)}{(z-1)^2 z^2} \alpha_1' - \frac{12-(z-1)z(\tau-8)(\tau+9)}{(z-1)^2 z^2} \alpha_1'' - 8 \left(\frac{1}{z} + \frac{1}{z-1} \right) \alpha_1'''$$

where we introduced the parameter $\tau = \frac{\sqrt{4\lambda+9\xi}-\sqrt{\xi}}{2\sqrt{\xi}}$. Obtaining this equation requires some elaborate algebraic manipulations. The calculations in this section were thus verified with the tensor algebra system xAct on Mathematica [11]. This equation can be solved in closed form. Using the asymptotic formulas in table I the solution verifying: i) regularity at $z = 0$ and ii) flat space short distance singularity is found. For non integer values of τ it reads

$$\alpha_1^{[\lambda,\tau]}(z) = \frac{2Q_\tau^2(2z-1) - \pi \cot(\pi\tau) P_\tau^2(2z-1)}{32\pi^2 R^2 \lambda (z-1) z (\tau^2 + \tau - 2)} + \frac{3\lambda - 1}{96\pi^2 R^2 \lambda} \left[\frac{(2z+1) \ln(1-z)}{z^2} + \frac{1}{z} \right] + \frac{1}{32\pi^2 R^2 \lambda (\tau-1)(\tau+2)} \left[\frac{1}{(z-1)^2} - \frac{(\lambda+1)\tau(\tau+1) - 2\lambda}{z-1} \right]$$

where P and Q are Legendre functions of the first and second kind respectively. As before, α_2 is obtained by (9). The asymptotic behavior in the IR is given by ($\tau > 1$):

$$\alpha_1^{[\lambda,\tau]}(z) \sim - \frac{2^{2\tau-5} \cot(\pi\tau) \Gamma(\tau + \frac{1}{2})}{\pi^{3/2} R^2 \lambda (\tau+2) \Gamma(\tau)} z^{\tau-2}$$

$$\alpha_2^{[\lambda,\tau]}(z) \sim \frac{2^{4\tau-1} (\tau-1) \tau \cot(\pi\tau) \Gamma(\tau + \frac{3}{2}) \Gamma(\tau + \frac{5}{2})}{\pi^2 R^2 \lambda (2\tau+3) (2\tau^2+5\tau+2) \Gamma(2\tau+2)} z^{\tau-1}$$

Thus for $\tau > 2$ for instance, the two-point function grows like $z^{\tau-2}$ in the IR. This strong IR growing is purely a gauge artifact since, as proven in the beginning of this letter, no IR pathologies arise in the Landau gauge.

	$z \rightarrow 0$	$z \rightarrow 1$	$z \rightarrow \infty$
$P_\tau^{(2)}(2z-1)$	$-\frac{\sin(\pi\tau)}{\pi z}$	$\frac{\Gamma(\tau+3)}{2\Gamma(\tau-1)}(z-1)$	$\frac{2^{2\tau}\Gamma(\tau+1/2)}{\sqrt{\pi}\Gamma(\tau-1)} z^\tau$
$Q_\tau^{(2)}(2z-1)$	$-\frac{\cos(\pi\tau)}{2z}$	$\frac{1}{2(z-1)}$	$\frac{\sqrt{\pi}\Gamma(\tau+3)}{2^{2(\tau+1)}\Gamma(\tau+3/2)} \frac{1}{z^{\tau+1}}$

TABLE I. Asymptotics of the Legendre functions near singular points.

A PHYSICAL DECOMPOSITION

The previously described situation, namely that the IR behavior of the two-point function in de Sitter space

contains important gauge artifacts makes it natural to look for a decomposition of the two-point function into a physical and a non-physical parts. We show now that such a decomposition exists and is given by the following rewriting of the two-point function

$$W_{aa'}^{[\lambda]}(x, x') = g_{aa'} \beta_1(z) + \nabla_a \nabla_{a'} \beta_2(z). \quad (13)$$

In fact, most of the physics is included in the “quantum” action W_J defined by

$$\exp \left[\frac{i}{\hbar} W_J \right] = \int D[A_a] \exp \left[\frac{i}{\hbar} \left(S + \int d^4x \sqrt{-g} A_a J^a \right) \right]$$

where J_a is an external conserved current $\nabla_a J^a = 0$. The tree-level expression of the effective action is given by

$$W_J = \int dV_x dV_{x'} J^a(x) W_{aa'}(x, x') J^{a'}(x')$$

where $dV_x = d^4x \sqrt{-g(x)}$ is the invariant volume element. Since the current J_a is conserved, integration by parts ensures that only β_1 will contribute to W_J and it thus fully deserves to be referred to as the “physical” two-point function. An explicit form of the functions β_1 and β_2 can be found by solving the system

$$\alpha_1 = \beta_1 + \frac{1}{2R^2} \beta_2', \quad R^2 \alpha_2 = (1-z) \beta_2' + z(1-z) \beta_2''.$$

The solutions are unique if one requires regularity near $z = 0$ and we obtain

$$W_{aa'}^{\text{Phys}}(x, x') = \frac{1}{16\pi^2 R^2} \left(\frac{1}{1-z} + \frac{\ln(1-z)}{z} \right) g_{aa'} \quad (14)$$

It is a remarkable (and expected) fact that $W_{aa'}^{\text{Phys}}$ is (λ, ξ) independent and is well behaved in the IR. In flat space-time, the Feynman gauge ($\lambda = 1, \xi = 0$) gives precisely this physical two-point function. The situation in de Sitter is quite different and no choice of the gauge fixing parameters gives the physical two-point function. A natural question is whether a more general gauge fixing term gives directly $W_{aa'}^{\text{Phys}}(x, x')$ in de Sitter space.

OUTLOOK

We showed that the IR pathological behavior of the photon’s two-point function is nothing but a gauge artifact. We suggest that the situation is similar for the graviton field. The IR behavior of the latter is however necessarily more intricate since it has been shown that the (transverse-traceless) growing term of the two-point function does not cancel out from scattering amplitudes [7, 10]. This implies in particular that a decomposition like (14) does not lead to a well-behaved “physical” two-point function. The IR behavior of the graviton field will be examined in [12].

ACKNOWLEDGMENTS

It is a pleasure to thank J.P Gazeau, A. Higuchi, K. Meissner, U. Moschella, E. Mottola, A.M. Polyakov and A. Starobinsky for stimulating discussions about de Sitter quantum field theory.

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